

M5 ANALYSIS OF A RADIAL FREE JET

1. Aim and practical aspects of the measurement

In ventilation systems, radial free jets are often used for low-speed air introduction. In radial jets, just like in round jets, the velocity of the flow reaches its maximum value in the plane of symmetry. As we are getting farther from the air-outlet section, the maximum value of the flow velocity is decreasing as the jet is getting wider and wider. The aim of the measurement is to get to know the properties of a radial jet by measuring its flow field.

2. Measurement setup

The measurement setup is illustrated in **Figure 1**. Air is sucked into the system from the through an inlet orifice which is suitable for measuring the volume flow rate of air entering the system from the surrounding air. The radial free jet is developed by a cylindrically symmetric nozzle (exhaust/outlet device). There are four different nozzles of $\beta = 30^\circ, 45^\circ, 60^\circ, 90^\circ$ angle relative to the axis of symmetry (see **Figure 1** below). The opening of the nozzle can be varied between $h_{min} = 1$ [mm] and $h_{max} = 4$ [mm]. Openings wider than h_{max} are not suggested, because it will cause an asymmetric flow pattern already at the outlet cross-section and the axis of the jet will greatly deviate from the geometrical axis given by β .

During the measurement, only *a single* slot height (h) and nozzle angle (β) must be investigated. The velocity field of the radial free jet can be mapped by measuring the dynamic pressure distribution using a Pitot probe and a digital manometer. The Pitot tube can be precisely positioned with the help of an adjustable stand.

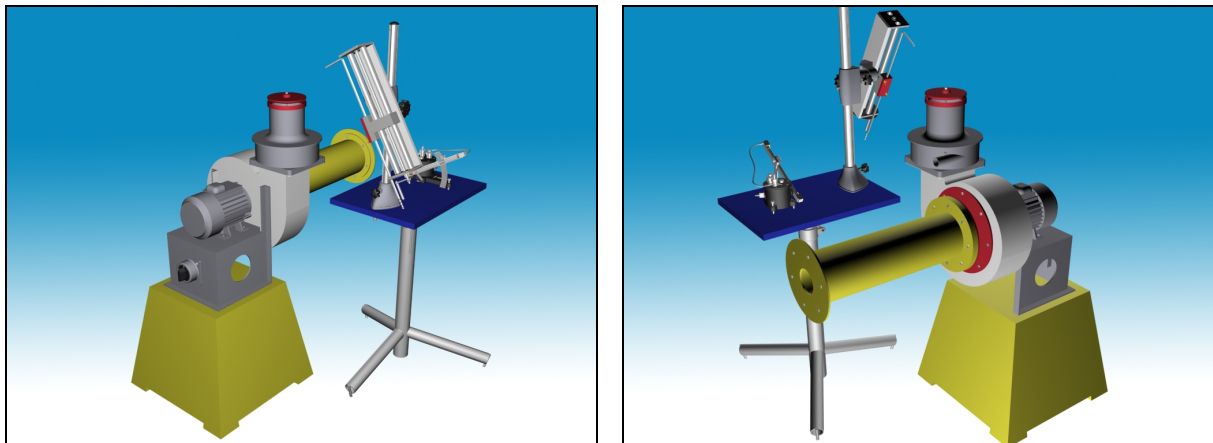
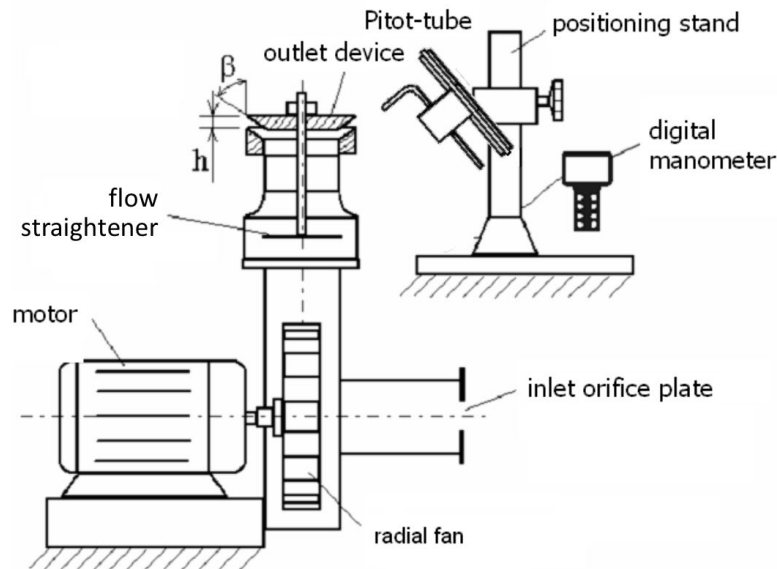


Figure 1. Schematic drawing and 3D renderings of the measurement setup.

3. Principles of the measurement

The static pressure inside free jets is approximately equal to the atmospheric pressure, therefore the difference of the total pressure measured using the Pitot probe (p_t) and the atmospheric pressure (p_0), both measured by the handheld digital manometer, gives us the dynamic pressure (p_{dyn}) of the flow at a given point, from which the velocity can be easily calculated.

Defined by the shape of the outlet device, the jet is either planar or truncated cone-shaped, and the streamlines are approximately parallel. As we move farther from the outlet orifice in the radial direction (r), the maximum velocity at a given $r = const.$ cross-section (v_{max}) is getting smaller, while the volume flow rate of the air flowing in the free jet (q_V) is getting larger, as the jet entrains air from the surrounding space (see [1] for further details).

4. Measurement procedure

- 4.1) Perform the calibration of the digital manometer via comparison to a Betz type manometer.
- 4.2) Measure the pressure drop on the inlet orifice plate necessary for the calculation of the inlet volume flow rate.

Measure the pressure difference between the atmospheric pressure and the pressure at the static pressure taps located after the orifice plate using the digital manometer. The volume flow rate can be calculated based on this value.

- 4.3) Check the symmetry of the radial free jet.

Set the nozzle width (slot height, h) according to your personal measurement task.

Measure the dynamic pressure of the air exiting the nozzle directly at the exhaust cross-section ($r = 0$) in four locations (0° , 90° , 180° , 270°) by holding the Pitot probe against the nozzle by hand. Determine whether the free jet is approximately symmetric or not.

- 4.4) Map the flow field of the radial free jet by measuring the lateral profiles of the velocity at different radial distances from the nozzle ($v(r,y)$).

Fix the Pitot tube onto the positioning stand in a way that the device is suitable for measuring the radial velocity exiting the nozzle.

Set the first measurement location for the Pitot probe with the help of the threaded rods of the positioning stand. The first measurement location should be right at the exhaust ($r = 0$) and at the symmetry axis ($y = 0$). For the interpretation of the coordinates, the Reader is referred to **Figure 2**.

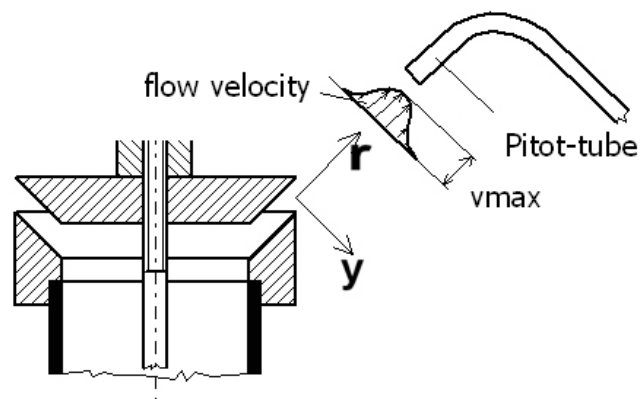


Figure 2. The coordinate system. The radial coordinate (r) is parallel to the direction of the flow exiting the nozzle, while the lateral coordinate (y) is perpendicular to this direction.

Measure half of the velocity profile in one direction (e.g., $y > 0$). Return to the axis of symmetry ($y = 0$) where it is not necessary to measure the value again, then perform the measurement of the velocity distribution in the other direction as well (e.g., $y < 0$).

Important: make sure that the measurement points are equidistant within one profile. Also, it is crucial that the measurement points reach both edges of the free jet ($p_{dyn,i} \leq 0$ at the last points).

Perform the measurement of the velocity profiles listed in your individual measurement task at all r radial distances. Based on the estimated value of the velocity gradient ($\Delta v/\Delta y$), apply different equidistant resolutions of the measurement grid, for example, 1 [mm] close to the exhaust and 2...3 [mm] in the far-field.

5. Post-processing and evaluation of the results

5.1) Perform the correction of the pressure values measured by the digital manometer based on the calibration of the device.

Prepare the calibration diagram based on the data pairs recorded with the Betz manometer and the digital manometer. Fit a straight trendline to the data and use its equation to correct all pressure values measured by the digital device.

$$p_{corr} = S \cdot p_{meas} + p_{offset}$$

In the above formula p_{meas} [Pa] is the measured pressure value, p_{corr} [Pa] is the corrected pressure value, S [-] is the slope of the calibration line, and p_{offset} [Pa] is the offset error of the device. If the device was zeroed before the measurement, then $p_{offset} = 0$.

5.2) Calculate the volume flow rate of air entering the system through the inlet orifice.

Provided that the pressure drop at the orifice is given, the volume flow rate can be calculated as:

$$q_{V,orifice} = \alpha \cdot \varepsilon \cdot \frac{d_{orifice}^2 \pi}{4} \cdot \sqrt{\frac{2}{\rho} \Delta p_{orifice}}$$

In the above formula $q_{V,orifice}$ [m³/s] is the volume flow rate through the orifice, $\alpha = 0.6$ [-] is the contraction coefficient, $\varepsilon = 1$ [-] is the expansibility factor, $d_{orifice}$ [m] is the diameter of the orifice, ρ [kg/m³] is the density of air, and $\Delta p_{orifice}$ [Pa] is the differential pressure measured across the orifice.

Air density can be calculated from the environmental data:

$$\rho = \frac{p_0}{RT_0},$$

in which p_0 [Pa] is the atmospheric pressure, $R = 287$ [J/(kg·K)] is the specific gas constant of air, and T_0 [K] is the room temperature.

5.3) Calculate the difference of the exhaust flow rate relative to the inlet flow rate.

Calculate the relative difference of the flow rate of the air exiting the system at the nozzle and the air entering the system at the orifice based on the below formula.

$$\Delta q_V = \frac{q_{V,out} - q_{V,in}}{q_{V,in}} \cdot 100 [\%]$$

The flow rate at the exhaust can be calculated as the product of the average of the four velocity values measured around the nozzle and the area of the exhaust:

$$q_{V,out} = v_{avg} \cdot D_{nozzle} h \pi$$

Evaluate the extent of the difference and explain the possible reasons in text.

5.4) Analyze the symmetry of the radial free jet.

Calculate the relative difference of the velocities measured at the 0°, 90°, 180° and 270° positions around the nozzle as:

$$\Delta v_{out} = \frac{v_{max} - v_{min}}{v_{avg}} \cdot 100 [\%]$$

State whether the deviation is small enough to consider the radial free jet cylindrically symmetric, therefore, to measure the velocity profiles only in the number of planes specified in the individual measurement task.

5.5) Plot all $v(y)$ velocity profiles measured at different r distances from the orifice in the same chart.

From the dynamic pressure measured at the i th measurement location, the pointwise velocity can be calculated as follows.

$$v_i = \sqrt{\frac{2}{\rho} p_{dyn,i}}$$

In the above formula v_i [m/s] is the local velocity, ρ [kg/m³] is the density of air, and $p_{dyn,i}$ [Pa] is the local dynamic pressure.

The velocity at the points outside the free jet (where $p_{dyn,i} \leq 0$ applies) should be considered zero.

5.6) Plot all dimensionless velocity profiles ($v/v_{max}(y/y_{1/2})$) measured at different r distances from the orifice in the same chart.

The velocity values should be normalized by the maximum velocity in the profile measured in that distance from the nozzle (v_{max}). The lateral coordinates should be normalized by the coordinate of the point where the velocity is reduced to half of the maximum value ($y_{1/2}$). To obtain the accurate $y_{1/2}$ value, use linear interpolation between the measurement points. Make sure that the distance corresponding to $v_{max}/2$ are found on both sides of the symmetric profile ($y_{1/2+}$ and $y_{1/2-}$), and the final $y_{1/2}$ value is obtained by averaging them.

5.7) Plot the maximum velocity of the profiles against the distance from the nozzle: $v_{max}(r)$.

5.8) Plot the volume flow rate of the air in the free jet against the distance from the nozzle: $q_V(r)$.

Important: the post processing of the results must include the brief evaluation of the results in text. Survey the technical literature on free jets (such as [1], but other sources can be cited as well), and compare the charts obtained from your measurement data with other results

about radial or planar free jets. Discuss the length of the initial section, analyze the self-similarity of the free jet, and compare the tendencies observed in your diagrams regarding the maximum velocity and the volume flow rate as the function of the distance from the nozzle to the formulas found in the literature, as accurately as possible.

Guidelines for the numerical integration:

In Task 5.8) the flow rates at different r distances can be obtained by simple numerical integration of the velocity profiles. The steps of the calculation are detailed below for $\beta = 90^\circ$, i.e., for perpendicular outflow.

- 1) The examined cross section of the free jet is a cylinder (see **Figure 3**), the numerical integration must be carried out on this surface.

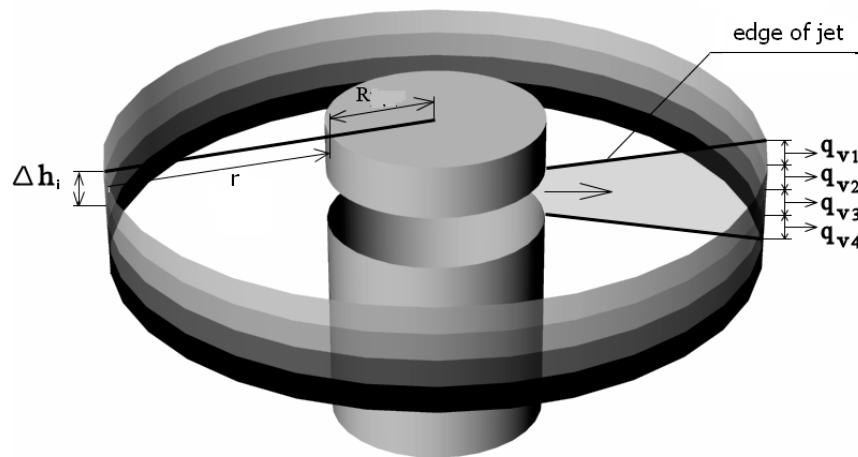


Figure 3. The flow rate can be obtained by the numerical integration of the velocity profile: $q_V = q_{V1} + q_{V2} + q_{V3} + q_{V4}$.

- 2) The area of the cylindrical sub-surface $A_{cyl,i}$ [m²] corresponding to the i th measurement point within the j th velocity profile can be calculated based on the radius of the nozzle (R [m]) and the distance between the exhaust and the current velocity profile (r_j [m]):

$$A_{cyl,i} = 2(R + r_j)\pi \cdot \Delta h_i,$$

in which Δh_i [m] is the width of the sub-surface corresponding to the i th measurement point. For an equidistant measurement grid, the value of Δh_i is equal to the distance of two neighboring measurement points (d_p).

- 3) The partial volume flow rate $q_{V,i}$ [m³/s] corresponding to the i th measurement point can be obtained as the product of the v_i [m/s] pointwise velocity and the corresponding sub-surface area:

$$q_{V,i} = v_i \cdot A_{cyl,i}$$

- 4) In a given cross section (in each velocity profile in different radial distances) the total volume flow rate can be calculated as the sum of the partial flow rates.

Important: the width of the free jet in a given r distance is equal to the total width of the sub-surfaces used for the numerical integration:

$$d_{jet} = \sum_{i=1}^n \Delta h_i = n \cdot \Delta h,$$

For an *equidistant resolution*, the total width is simply the product of the number of sub-surfaces n [–] and the sub-surface width (Δh [m]). It should be noted, however, that the width obtained this way (d_1 in **Figure 4**) is *not* equal to the distance of the two farthest measurement points of non-zero velocity (d_2), but it is larger by a whole step!

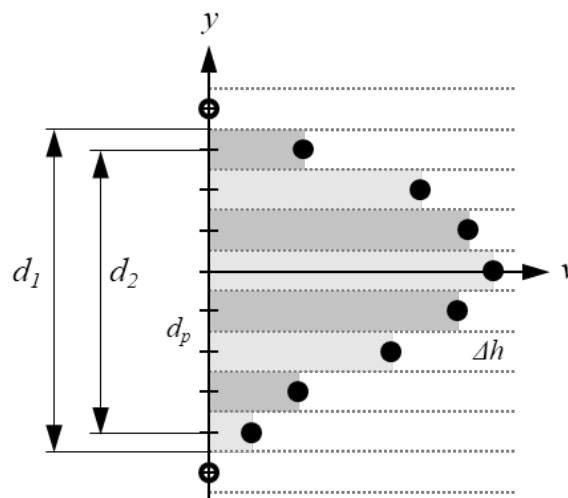


Figure 4. For an equidistant measurement grid, the distance between neighboring points (d_p) and the width of the sub-surfaces (Δh) used for the numerical integration of the velocity profile is equal in value. Note that there is a difference of a whole step size between the width of the free jet (d_1) and the distance of the farthest measurement points inside the free jet (d_2).

Remark: the formula for numerical integration of the velocity profile at the j th distance for an angled ($\beta \neq 90^\circ$) nozzle would be:

$$q_{V,i} = A_{cyl,i} \cdot v_i = \sum_{i=1}^n 2(R + r_j \cdot \sin \beta) \cdot \pi \cdot \Delta h_i \cdot v_i$$

6. Measurement uncertainty calculation

The measurement uncertainty of all points of a chosen velocity profile outside the initial section of the free jet should be assessed. Based on the law of Gaussian error propagation, the total measurement uncertainty of a quantity (δR) depends on the measurement uncertainties of k pieces of independent quantities (δX_k) as follows:

$$\delta R = \sqrt{\sum_{k=1}^n \left(\delta X_k \cdot \frac{\partial R}{\partial X_k} \right)^2}$$

The measurement error should be calculated for the pointwise velocities, therefore $R = v_i$.

During the experiment, the following directly measured quantities (X_k) are subjected to measurement error (δX_k):

- | | | |
|-----------------------------------|-------------------|--|
| ➤ Dynamic pressure (Pitot probe): | $X_1 = p_{din,i}$ | $\delta X_1 = \delta p_{din,i} = 2$ [Pa] |
| ➤ Atmospheric pressure: | $X_2 = p_0$ | $\delta X_2 = \delta p_0 = 100$ [Pa] |
| ➤ Room temperature: | $X_3 = T_0$ | $\delta X_3 = \delta T_0 = 1$ [K] |

6.1) Substitute the applied formulas to the expression of the v_i pointwise velocity (5.5) until it contains only directly measured quantities and known constants.

Show the final expression along with a brief explanation.

6.2) Calculate the $\partial v_i / \partial X_k$ quantities by partially differentiating the above final formula by the directly measured quantities.

Show the formulas for the three partial derivatives along with a brief explanation.

6.3) Calculate the absolute (δv_i) and relative ($\delta v_i / v_i$) measurement uncertainty of the pointwise velocity for all points in the profile by substituting to the formula of the Gaussian uncertainty propagation.

Plot the absolute and relative error against the lateral coordinate (y) in separate diagrams. Show the magnitude of the absolute measurement error plotted as error bars on the original velocity profile. Discuss the results in text.

Remarks

Be aware of the following during the measurement:

- Before turning any measurement device on or in general during the lab, make sure that safe working conditions are ensured. The other participants have to be warned of the starting of the machines and of any changes that could endanger the members of the lab.
- The atmospheric pressure and room temperature should be recorded before and after every measurement.
- The measurement units and other important factors (e.g. data sampling frequency, date of calibration) of every recorded value of the applied measurement devices should be recorded.
- Type and construction number of the applied measuring instrument should be included in the final report.
- The geometrical parameters to be used in the calculations must be measured.
- Checking and harmonizing of the units of the recorded values with those used in further calculations.
- Manometers should be calibrated.
- The measurement ports of the pressure meter should be carefully connected to the correct pressure ports of the measurement instrumentation. The integrity of the silicone

tubes used for connection must be verified before and during the measurement (no holes, slits, etc.).

- If inlet or outlet tubes are to be assembled with fans, connections should be airtight as escaping/entering air can significantly modify the measurement results.

Before submitting the lab report, it is necessary to check whether its *structure, style and visual appearance* meets the requirements (the present Measurement Guide is also a good example). It is recommended to ask for a *consultation* opportunity from the Measurement Responsible before submission if you have any questions.

The list of requirements, an example for the lab report, as well as an uncertainty calculation guide can be found at [this](#) link. Animations about the measurement can be accessed through [this](#) link.

Avoiding even the suspicion of *plagiarism* concerning the lab report is crucial. Therefore, taking any written or visual content (text, images, etc.) of another work – including the present Measurement Guide – must be indicated by a *proper reference* to the original work. Ideas taken from another source should be rephrased in the lab report using your own words, proving your own understanding of the contents.

References

- [1] Lajos Tamás: Az áramlástan alapjai. 7.5. lecke: Szabadsugarak. Műegyetemi Kiadó, 2004.

Remark: this is a Hungarian book on Fluid Dynamics. The chapter on free jets supplemented with English comments can be acquired from the Measurement Responsible upon request. It is not mandatory to use this as a reference for the comparison of the measurement results to the literature; other trustworthy sources can be cited alternatively.